Asymptotic Behaviour of Platoon Systems

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***Abstract*— In this paper we study the asymptotic behaviour of various platoon-type systems using the general theory developed by the authors in a recent article. The aim is to steer an infinite number of vehicles towards a target configuration in which each vehicle has a prescribed separation from its neighbour and all vehicles are moving at a given velocity. More specifically, we study systems in which state feedback is possible, systems in which observer-based dynamic output feedback is required, and also a situation in which the control objective is modified to allow the target separations to depend on the vehicles' velocities. We show that in the first and third cases the objective can be achieved, but that in the second case the system is unstable in the sense that the associated semigroup is not uniformly bounded. We also present some quantified results concerning the rate of convergence of the platoon to its limit state when the limit exists.**

***Index Terms*— Vehicle platoon, system, ordinary differential equations, asymptotic behaviour, control, adaptive control, state feedback, rates of convergence**

I. Introduction

The purpose of this paper is to study dynamic properties the so-called *platoon system* [1–5], which describes the behaviour of an infinite chain of vehicles on a highway. The main objective is to ensure that the distances between the vehicles converge asymptotically to given target values. The behaviour of the full system is described by ordinary differential equations of the form

$$\dot{x}_k(t) = A_0 x_k(t) + A_1 x_{k-1}(t), \qquad k \in \mathbb{Z},\ t \geq 0, \tag{1}$$

where A_0 and A_1 are $m \times m$ matrices for some $m \in \mathbb{N}$ and where the initial states $x_k(0) \in \mathbb{C}^m$ for $k \in \mathbb{Z}$ are known.

The exact forms of the matrices A_0 and A_1 depend on the formulation of the control problem for the platoon system. In this paper we consider the following three different versions:

(i) In the first version we assume that state feedback can be employed in the control of the individual vehicles.
(ii) In the second version we assume that the states of the vehicles are not available for feedback, and we instead use observer-based dynamic output feedback in the control of the vehicles.
(iii) In the third version we consider a modified control objective employing a so-called *constant headway time policy* in which the target distances between the vehicles depend on the velocities of the vehicles.

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The system (1) can be written as an abstract linear differential equation

$$\dot{x}(t) = Ax(t), \qquad x(0) = x_0 \in X \tag{2}$$

on the infinite-dimensional state space $X = \ell^\infty(\mathbb{C}^m)$. The system operator A is a bounded linear operator defined by $Ax = (A_0 x_k + A_1 x_{k-1})_{k\in\mathbb{Z}}$ for all $x = (x_k)_{k\in\mathbb{Z}} \in X$. Each of the situations (i)–(iii) can be formulated in such a way that the separations between the vehicles converge to appropriate target distances if the solutions $x(t)$, $t \geq 0$, of (2) decay to zero asymptotically, i.e., $x(t) \to 0$ as $t \to \infty$ for all $x_0 \in X$. The main purpose of this paper is to present conditions for the convergence of the solutions of (2) as $t \to \infty$. In addition we are interested in the rate of the convergence. Our results are based on application of recent theory for a more general class of infinite systems of differential equations presented in [6] and on recent developments in the theory for asymptotic behaviour of strongly continuous semigroups [7–9].

In the situation (i) we investigate the behaviour of the full platoon system under suitable stabilising state feedback control in the individual vehicles. We characterise the spectrum of A and show that semigroup generated by the system operator A is uniformly bounded. We also characterise the initial states of the full system that lead to convergent solutions and show that under additional conditions the convergence $x(t) \to z$ as $t \to \infty$ happens at a particular rational rate. In the earlier references the platoon model has been studied on the state space $X = \ell^2(\mathbb{C}^m)$, and it has in particular been shown that the system is not exponentially stabilisable [2], [5] but that strong stability can be achieved [2], [10]. Our results characterise the asymptotic behaviour of the full system on the space $X = \ell^\infty(\mathbb{C}^m)$ which can be argued to be a more realistic choice for a state space [2]. In particular, our results demonstrate that the behaviour of the platoon system on the spaces $\ell^2(\mathbb{C}^m)$ and $\ell^\infty(\mathbb{C}^m)$ differs in the respect that on the latter space some solutions do not converge at all and some of them converge to nonzero final states. This is in contrast to the fact that on $\ell^2(\mathbb{C}^m)$ the corresponding system (2) is strongly stable and all solutions satisfy $x(t) \to 0$ as $t \to \infty$.

In the situation (ii) we use identical observer-based output feedbacks to study the dynamics of the individual vehicles. We prove that regardless of the choice of the observer parameters, the system will be unstable and in particular some of the solutions $x(t)$, $t \geq 0$, of (2) will diverge at exponential rates.

Finally, in the situation (iii) the requirement for the convergence of the distances to static values is replaced by the calculation shows that $n_\phi = 2$. Since $\phi(0) = 1$ and

$$A_1 A_0^{-1} = \frac{1}{h}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & -\tau & -h \end{pmatrix},$$

$$A_0^{-1} A_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

the claims of the theorem follow from Theorem 2.5. ■

Remark 5.2: Our more recent results show that if $\alpha_0, \alpha_1, \alpha_2 > 0$ are chosen in such a way that $\sigma(A_0) = \{-1/h\}$, then the logarithms in parts (i) and (iii) of Theorem 5.1 can be omitted. The decay rates then become $O(t^{-1/2})$.

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